B.Sc. Degree II 4th Paper

Diff. Equations

Linear Diff. Equations with constant coefficients

Q: Solve $(D^2+9)y = x\sin x$.

Soln

For CF, $D^2+9=0 \Rightarrow D^2 = 9i^2$

$\Rightarrow D = \pm 3i$

$\therefore CF = c_1\cos 3x + c_2 \sin 3x$.

Now, for PI, $y = \frac{1}{D^2+9} x\sin x$

$\Rightarrow y = \text{I.P. of } \frac{1}{D^2+9} x(\cos x + i\sin x)$

$\Rightarrow y = \text{IP of } \frac{1}{D^2+9} x e^{ix}$

$\Rightarrow y = \text{IP of } e^{ix} \frac{1}{(D+i)^2+9} x = \text{IP of } e^{ix} \frac{1}{D^2+2Di+8} x$

$$\Rightarrow y = \text{IP of } e^{ix} \frac{1}{D^2 + 2iD + 8} x$$

$$\Rightarrow y = \text{IP of } e^{ix} \frac{1}{8\left[1 + \frac{D^2 + 2iD}{8}\right]} x$$

$$\Rightarrow y = \text{IP of } \frac{e^{ix}}{8} \left(1 + \frac{D^2 + 2iD}{8}\right)^{-1} x$$

$$\Rightarrow y = \text{IP of } \frac{e^{ix}}{8} \left[1 - \frac{D^2 + 2iD}{8} + \text{higher powers of } D\right] x$$

$$\Rightarrow y = \text{IP of } \frac{e^{ix}}{8} \left[x - \frac{D^2(x) + 2iD(x)}{8} + \text{higher powers of } D(x)\right]$$

$$= \text{IP of } \frac{e^{ix}}{8} \left[x - \frac{0 + 2i.1}{8} + 0\right]$$

$$= \text{IP of } \left(\frac{\cos x + i \sin x}{8}\right) \left(x - \frac{i}{4}\right)$$

$$= \text{IP of } \frac{1}{32} (\cos x + i \sin x)(4x - i)$$

$$\Rightarrow PI = \frac{1}{32}(-\cos x + 4x \sin x)$$

Hence the complete soln is $y = CF + PI$